AIAA JOURNAL
Vol. 34, No. 7, July 1996

Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "New Similarity Solutions for Hypersonic Boundary Layers with Application to Inlet Flows," Part 1

Jack Pike*
Chawston, Bedfordshire MK44 3BH, England, United Kingdom

IN this paper[1] two new self-similar solutions for two-dimensional laminar hypersonic boundary layers are derived to complement previous well-known solutions.[2] A feature of the new solutions is that the self-similar body shapes are preceded by a plain ramp, on which, presumably, a boundary layer develops upstream of the body, which generates a power law or exponential self-similar boundary layer. It is necessary at the junction between the ramp and the curved body that the boundary layer is continuous and hence that the boundary layer generated by the ramp is a solution of the downstream self-similar curved body boundary layer. That is, the distributions of density, velocity, and entropy across the boundary layer at the junction must be the same for both the ramp and curved body self-similar solutions.

The transformed y coordinate across the boundary layer is given by Inger's[1] Eq. (1) to be

$$\eta = \frac{U_e}{G}\int_0^y \rho\,\mathrm{d}y \tag{1}$$

where U_e is the velocity at the edge of the boundary layer and G is a function of x found as part of the solution. This y transformation can be made the same for both boundary-layer solutions at the junction, by choosing appropriate values of arbitrary constants that appear in the solutions for G, to make the value of G at the junction the same for both solutions.

Based on this common y coordinate transformation, the variation of the velocity and entropy across the boundary layer must be the same. For self-similar solutions, these are given by Inger's Eqs. (14) and (15) to be

$$ff'' + f''' = \lambda(1 - 1/\gamma)(g - f'^2) \tag{2}$$

$$fg' + g'' = 0 \tag{3}$$

where

$$\frac{u}{u_e} = f'(\eta) = \frac{\mathrm{d}f}{\mathrm{d}\eta} \tag{4}$$

$$H/H_e = g(\eta) \tag{5}$$

The boundary layer on the plain ramp can be described by the self-similar solution of Eqs. (2–5) by putting $\lambda = 0$ in Eq. (2). The boundary layer on the power law or exponential downstream body is described by $\lambda(1 - 1/\gamma)$ being a nonzero constant (for a curved body). The variation of f and g with η given by Eqs. (2) and (3) depends on the value of this constant and differs for different values of the constant. This is illustrated, for example, in Inger's Fig. 3, where a change in β_{CR} [$= -\lambda(1 - 1/\lambda)$] results in a change in the shear function at the wall.

Hence the variation of f and g with η at the junction where $\lambda(1 - 1/\lambda)$ changes value is not continuous for the two self-similar solutions, and the initial boundary layer at the start of the curved body will not have the correct self-similar form. Thus, certainly near the beginning of the curved self-similar region, the boundary-layer distribution and other dependent boundary-layer values must be expected to differ from the self-similar analytically derived expressions of Ref. 1.

Received Jan. 20, 1996; accepted for publication April 2, 1996.

*Independent Consultant, Holly Cottage. Member AIAA.

References

[1]Inger, G. R., "New Similarity Solutions for Hypersonic Boundary Layers with Application to Inlet Flows," *AIAA Journal*, Vol. 33, No. 11, 1995, pp. 2080–2086.

[2]Mirels, H., "Hypersonic Flow over Slender Bodies with Power Law Shocks," *Advances in Applied Mechanics VII*, Academic, New York, 1962, pp. 1–54.

Comment on "New Similarity Solutions for Hypersonic Boundary Layers with Application to Inlet Flows," Part 2

Jack Pike*
Chawston, Bedfordshire MK44 3BH, England, United Kingdom

IN the paper by Inger,[1] two new self-similar solutions to the two-dimensional laminar hypersonic boundary-layer equations are found, which complement the previous two well-known solutions.[2] All of these solutions are power law or exponential solutions valid for particular x coordinate transformations $\xi = \xi(x)$. Solution cases 1 and 3 of Inger's paper are valid when $\xi = x$ and cases 2 and 4 when ξ is a function of

$$\int p_e\,\mathrm{d}x$$

This separation into various cases[1] for various ξ is shown here to be unnecessary, for they are all examples of a general solution valid for all ξ.

To obtain solutions that are self-similar, the coefficients of the terms in $f(\eta)$ and $g(\eta)$ in Eqs. (5), (6), and (9) of Ref. 1 need to be constants. These equations can be written as

$$\left(\frac{\mathrm{d}G/\mathrm{d}\xi}{G}\right)ff'' + K_G f''' = \left(\frac{\gamma - 1}{2\gamma}\right)\frac{(\mathrm{d}p_e/\mathrm{d}\xi)}{p_e}(g - f'^2) \tag{1}$$

Received Jan. 20, 1996; accepted for publication April 2, 1996.

*Independent Consultant, Holly Cottage. Member AIAA.